Why should the government provide the infrastructure through the Public-Private Partnership mode?

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Abstract: This paper develops an endogenous growth model with the non-rival but excludable public goods. We seek to answer the question that whether this kind of infrastructure should be provided by a pure private firm or purely by state or by Public-Private Partnership (PPP). In this paper, PPP in infrastructure is defined as a profit-making private firm-producing infrastructure with the partial cost borne by the government. If the government invests in this type of infrastructure, how should it finance the manufacturing cost - through accumulating debt or imposing a tax or by charging user-fees? We make a comparison of the macro-economic performances under the purely private provision, purely public provision and PPP provision of infrastructure in an economy.

Also, we study the public and PPP provision under different budgetary regimes of the government: (a) Zero debt (b) Constant debt and (c) accumulating debt regimes. In the purely public provision of infrastructure, if the government runs a balanced budget or has constant debt, our model suggests that government should finance the infrastructure by charging user fees instead of imposing the tax since the growth maximizing tax rate is zero. We also compare the maximum growth rates and user fees under different modes of provisions and across different budgetary regimes of the government. We find that, for countries with lower output elasticity of infrastructure capital and higher share of cost is borne by the infrastructure-manufacturing firm, then debt financing is desirable. Under the accumulating debt regime the user fee of infrastructure services under the PPP mode of provision is found out to be less as compared to the user fees of the public mode of provision in the accumulating debt regime.

Keywords: *Infrastructure, Public-Private Partnership, Endogenous growth, Public debt*

JEL Classification**: E62, H44, O40**

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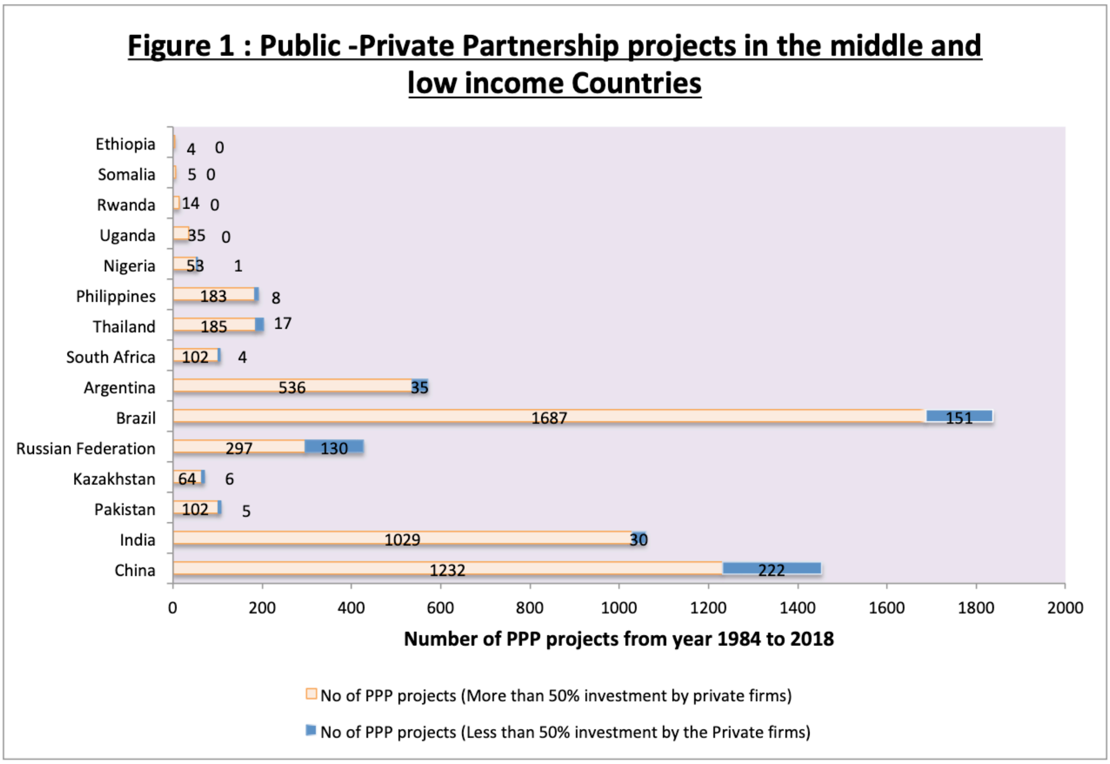
1. Introduction:

Traditionally, the government has been the unique provider of public goods and services in most of the developing nations. Much of the literature on infrastructure and endogenous growth theory concentrates on the non-rival and non-excludable pure public goods. To name a few, Barro (1990), Futagami et al. (1993), Dasgupta (1999), Turnovsky and Pintea (2006) and Bhattacharya (2014) study the non-rival and non-excludable publicly provided infrastructure and they find the optimal fiscal policy in a balanced budget framework. Though infrastructure services is an important development tool that catalyzes growth in the long run, it is difficult for the governments of the low-income developing countries to bear the entire enormous fund required for the cost of construction of infrastructure. Therefore, Public-Private Partnership (PPP) in infrastructure provision emerges as a solution to the problem of infrastructure crunch. In PPP, the government makes payment for a part of the total up-front cost and the rest of the construction phase's cost is taken care by the private firm and therefore government pays little or nothing throughout the infrastructure project. Usually, in PPP, there is a contract between a private firm and the government for a time period, and the management responsibility is of the private firm and they earn the profit revenue from the projects. Usually, the contract period is in the range of 10-50 or more number of years after which the ownership is transferred from the private firm to the government.

The government gets the political credit for delivering the project in the current period and has the advantage of improving the current budgetary position and minimizing the government deficit. The complete privatization is different from the PPP, the former has no direct government role in the ongoing operations of the project and the private firm has the monopoly status with little or no regulation, whereas in case of PPP the government retains the share of responsibility for investment as well as for the operational function when it is handed over the ownership by the manufacturing firm after it has made its profit over the years. On the other hand, in contrast to pure public provision of infrastructure, PPP provision reduces the need for high current taxation, reduces the financial cost on part of the government and therefore unbinds the public spending to other sectors. In recent years many developed and developing nations have adopted the Public-Private Partnership (PPP) in the provisioning of infrastructure services. Metro rail system of New Delhi, India; roads in Chile, Argentina, United States of America, Hong Kong, Hungary and Italy; water system of Singapore, Airports of New Delhi and Mumbai of India; rural electrification of Guatemala; port expansion in Colombo, Sri Lanka, etc; are some examples of successful PPP projects among many PPP projects investment taking place around the world. Figure 1 depicts the PPP projects undertaken in different middle and low-income countries from the year 1984-2018.

In Figure 1, we observe that the number of PPP projects undertaken by Brazil, China, India, Argentina and Russia are quite high and it is observed that the number of PPP projects under the category of more than 50% investment by private firms are quite high and numbers of PPP projects under the category of less than 50% investment by the private firms are few. The World Bank in its PPI (Private Participation in Infrastructure) Annual Report 2017 reported China (73 projects worth US dollar 17.5 billion), Brazil (24 projects worth US dollar 7.3 billion) and Pakistan (4 projects worth US dollar 5.9 billion) among the top 5 PPI investment countries.



Source: Author's own compilation from Private participation in Infrastructure Database, World Bank.

The World Bank's PPI Annual Report 2018 again reported China to be the leader in PPP projects with 37 projects worth US dollar 11.6 billion. Also, India (24 projects worth US dollar 3.8 billion) and Brazil (11 projects worth US dollar 3 billion) were featured among the top 5 PPI investment countries in 2018. The World Bank PPI Report noted that SAR (India, Pakistan) and ECA (Kazakhstan, Russian Federation, Turkey) countries have a sizable portion of public debt financing with 27% and 26% respectively. The lower-income groups of African countries have only a few PPP projects only after 1998. Most of the middle-income countries show the up-rise in the investment of PPP projects after 1991.

While going through the related literature, we find that there exists small literature dealing with a comparative evaluation of different mode of provisioning infrastructure. The model by Barro (1990) has been extended by Kamaiguchi and

Tamai (2012) by integrating public deficit and public debt into the model to analyze the conditions for simultaneous growth and sustainability of public debt.

Chatterjee and Morshed (2011) compare the impact of private and government provision of infrastructure on an economy's aggregate performance. Barro and Sala-i-Martin (1992), Futagami et. al. (1993), Fisher and Turnovsky (1998), Devarajan et. al. (1998) study the interaction between public and private capital in an endogenous growth context, where the public good is non-excludable. However, excludability feature of the public good is ignored in the above studies. A study by Ott and Turnovsky (2006) make a comparative study of the non-excludable and an excludable public good. Government is the unique supplier of public input and fixes the prices of the user fees. According to them, tax plus user fees alone could be sufficient to finance for the provision of the entire infrastructure. However, this kind of financing is possible only in the case of developed nations like the United Kingdom, the United States and EU nations but for developing nations like India, only user fee cannot alone suffice the financing of the excludable public goods. Kateja (2012), suggests that private partnership in infrastructure along with public investment offers significant advantages in terms of enhancing efficiency through competition in the provision of services to users. We see that privately provided roads, power, water, transportation, communication and irrigation etc; are quite common in the developed nations. However, in the developing nations, there is a dearth of private investors and the government fails to attract private investments. So, the governments of developing countries must use the policy instruments such as subsidy, tax holiday and sharing of the partial cost of manufacturing public good, which is called Viability Gap Funding (VGF). In India, the government under the VGF bears 20%-40% of the manufacturing cost of the infrastructure investment. Therefore, VGF could be an important policy tool for infrastructure provision in developing nations.

In this context, the research questions addressed in this paper is that, 'Is PPP mode of infrastructure provision a better option compared to the private and public mode of provision?' We investigate the reason behind why the government cannot produce the impure public goods, which are non-rival but excludable and charge the user fees itself. Why does it need the help of the private firm for the manufacturing of infrastructure? In this paper, we attempt to address primarily two questions: why should the government go for PPP for infrastructure provision? And how should the government finance the cost of infrastructure production - through the imposition of the tax, through bond financing or through charging only user-fees? We build a closed economy model of infrastructure provision to answer these questions. In this model, infrastructure may be provided by the pure private firm, pure public entity or through the partnership of private firm and public entity (PPP). We are considering different possibilities: the government may run a balanced budget or may have a budget deficit. In case of a budget deficit, there are two possibilities: the government may have constant debt or may accumulate debt over time. In the real world, the government borrows from the capital market, banks and issues public bonds to finance the public investments. Following, Greiner (2008, 2012) and Kamaiguchi and Tamai (2012), we assume that while the government runs a deficit budget, it must set the primary surplus as a positive linear function of public debt, which guarantees that the public debt is sustainable. Greiner (2008), compares the government's balanced budget outcome in terms of growth and welfare with the scenario when public debt grows at a lower rate than the balanced growth rate and also the scenario when public debt grows at the same rate as all other variables. However, Greiner (2008) study the government's different budgetary regimes under the pure public provision only and does not consider the infrastructure provision through the PPP mode or the private mode. According to him, not the constant debt but the declining debt ratio, in the long run, benefits the economy. Bara and Chakraborty (2019), study the optimality of PPP in an endogenous growth model and they treat the public capital and private capital as a substitute and as a complementary goods. However, their paper deals only with the balanced budget case and ignores the debt-financing situation. The present paper attempts to find whether the PPP mode under the different budgetary regime for infrastructure provision is optimal or not, as compared to the pure public provision and pure private provision. In the purely public provision of infrastructure, if the government runs a balanced budget or has constant debt, our model suggests that government should finance the infrastructure solely by charging user fees instead of imposing the tax since the growth maximizing tax rate is found out to be zero. The maximum growth rate possible to obtain under the balanced budget for the pure public provision of infrastructure is equal to the growth rate under the pure private provision of infrastructure. The economic growth rate and the user-fee charged are the same under zero debt and constant debt and this result is not surprising because the debt to capital ratio is zero in the long run in positive, constant debt regime. In zero debt regime, if output elasticity of infrastructure is high and the share of the cost borne by the government for infrastructure provision is high (in other words, the share of the cost borne by the private firm is low) then the PPP provision yield higher growth rate as compared to pure public/pure private provision. This result implies that PPP provision in the balanced budget regime may not be desirable, instead PPP provision in the permanent deficit regime is. For countries with lower output elasticity of infrastructure and higher share of cost is borne by the infrastructure-manufacturing firm, then debt financing is desirable.

The structure of the paper is organized in the following manner. Section 2 describes the competitive economy model of pure private infrastructure provision, pure public infrastructure provision and the PPP provision of infrastructure, where both the private firm and the government jointly provide the infrastructure. In subsections 2.2 and 2.3 we study the steady-state balanced growth rates for the zero debt regime, constant debt regime and the permanent deficit regime for pure public provision and PPP provision of infrastructure respectively. In Section 3, we describe the condition under which the PPP investment in infrastructure may be desirable and we also explain whether debt financing for the provision of infrastructure is justified or not? Section 5 concludes.

2. The model

We consider an economy with an infinitely lived representative agent. It is assumed that the household derives utility from direct consumption of final goods. The utility function of the representative household is given by,

$$U = \int_0^\infty (\gamma \ln C)\, e^{-\rho t} dt \tag{1}$$

ρ is constant and denotes positive discount rate at which future utility is discounted. In this closed competitive economy model, we have three agents - the representative household and two profit-making firms. We assume that there are two profit-making firms in the economy. Firm 1 produces final goods for consumption and capital accumulation. Firm 2 produces the infrastructure services. Following Barro (1990), we assume that infrastructure services are flow in nature. Both the firms are run by profit-maximizing private entities. Infrastructure is used for final goods production.

The household supplies private physical capital (K) required for the production of both final goods (Y) and infrastructure (G). The production function of final goods is given as,

$$Y = A(uK)^{1-\alpha}G^{\alpha} \quad , \qquad 0 < \alpha < 1 \tag{2}$$

Y is used for consumption as well as capital accumulation. u fraction of private physical capital is used for production of Y. Flow of infrastructure goods at time t is denoted by G. A is the technology parameter of production of Y. $1-\alpha$ and α are output elasticities with respect to uK and G respectively. A final goods is assumed to be a numeraire commodity. Hence, the price of Y is considered to be unity.

The production function of infrastructure service is given as,

$$G = \delta(1-u)K \tag{3}$$

In equation (3), $(1-u)K$ denotes the remaining part of private physical capital that is used to produce infrastructure. δ is the technology parameter of infrastructure services production, which is a constant. The infrastructure services are productive investment and are flow in nature. We obtain the ratio of infrastructure to private physical capital from equation (3),

$$\frac{G}{K} = \delta(1-u) \tag{4}$$

The profit function of firm 1 producing the final goods is given by equation (5).

$$\Pi^{1} = A(uK)^{1-\alpha}G^{\alpha} - ruK - \mu G \tag{5}$$

μ is the user fees or price paid by firm 1 for using infrastructure services. Firm 1 takes input prices as given and choose input quantities so as to maximize it's profit. Differentiating profit function of firm 1 with respect to uK and G, we obtain the following first- order condition equations are obtained:

$$r = \frac{A(1-\alpha)}{u^{\alpha}}\left(\frac{G}{K}\right)^{\alpha} \tag{6}$$

Substituting the value of $\left(\frac{G}{K}\right)$ in the above equation,

$$r = A(1-\alpha)\delta^{\alpha}\left(\frac{1-u}{u}\right)^{\alpha} \tag{7}$$

$$\mu = Au^{1-\alpha}\alpha\left(\frac{G}{K}\right)^{\alpha-1} \tag{8}$$

Substituting the value of $\left(\frac{G}{K}\right)$ in the above equation,

$$\mu = A\alpha\delta^{\alpha-1}\left(\frac{1-u}{u}\right)^{\alpha-1} \tag{9}$$

Infrastructure demand of firm 1 is obtained from equation (8),

$$G = \left[\frac{\mu}{Au^{1-\alpha}\alpha}\right]^{\frac{1}{\alpha-1}} K \tag{10}$$

Equating the supply function of infrastructure with the demand function of infrastructure, we obtain the equilibrium allocation of private physical capital between final goods production and infrastructure production.

$$u = \frac{(A\alpha)^{\frac{1}{\alpha-1}}\delta}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta} \tag{11}$$

Substituting the value of u in equation (7), we obtain the rate of interest.

$$r = A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \tag{12}$$

2.1 Pure private provision of infrastructure

In the pure private provision of infrastructure, the profit-making private firm is responsible for providing the infrastructure services and hence it bears the entire manufacturing cost of the infrastructure and therefore, charges user fees. The government does not play any role in the provision of infrastructure. The profit function of infrastructure producing firm 2 is given by equation (13).

$$\Pi^2 = \mu G - r(1-u)K \tag{13}$$

In equation (13), μ is the user fee charged by firm 2 for the infrastructure services (for example; the user fees charged for the usage of roads). Firm 2 also takes input prices as given and chooses input quantities so as to maximize its profit. Differentiating the profit function of firm 2 with respect to $(1-u)K$, we obtain the first order condition.

$$r = \mu\delta \tag{14}$$

Now, equating the rate of interest of firm 1 and firm 2, we find the equilibrium value of u under the pure private provision of infrastructure, by no arbitrage condition.

$$u = 1 - \alpha \tag{15}$$

Now again substituting the value of u in equation (9), we obtain the user fee to be charged under the pure private provision.

$$\mu = A\delta^{\alpha-1}\alpha^{\alpha}(1-\alpha)^{1-\alpha} \tag{16}$$

2.1.1 The Households sector

The utility function of the representative household is given in equation (1). It is assumed that the households accumulate the disposable income over expenditure as wealth. The total wealth/ asset (W) of the households is equal to the total private physical capital (K) in the economy. Therefore, $W = K$. The rate of accumulation of private physical capital is given by (17).

$$\dot{K} = rK - C \tag{17}$$

The rate of growth of private physical capital is, obtained using equation (17).

$$\frac{\dot{K}}{K} = r - \frac{C}{K} \tag{18}$$

In competitive economy, the representative household maximizes the current-value Hamiltonian, subject to equation (17).

$$H_c = \gamma \ln C + \eta[rK - C] \tag{19}$$

The control variable is C. The first – order maximization condition is given in equation (20).

$$\frac{\gamma}{C} = \eta \quad (20)$$

The time-derivative of the co-state variable is given by the following equation.

$$\frac{\dot{\eta}}{\eta} = \rho - r \quad (21)$$

Taking the log and derivative of equation (20), we get the growth rate equation under the pure private provision.

$$-\frac{\dot{C}}{C} = \frac{\dot{\eta}}{\eta} = \rho - r$$

Hence, the growth rate under the pure private provision is given by equation (22).

$$\frac{\dot{C}}{C} = r - \rho = g \quad (22)$$

2.1.2 Steady State Balanced growth

At steady state balanced growth rate under the pure private provision, $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = g$. If $\frac{\dot{K}}{K}$ is constant, $\frac{C}{K}$ is also constant. The steady state growth rate, g is constant and positive. Now setting $\frac{\dot{C}}{C} = \frac{\dot{K}}{K}$, we get,

$$\frac{C}{K} = \rho \quad (23)$$

Substituting the value of r from equation (14) and the value of user fees from equation (16) in the growth rate equation (22), we obtain the steady state balanced growth rate under the pure private provision.

$$g = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\delta^{\alpha} - \rho \quad (24)$$

Proposition 1: *There exists a unique steady state balanced growth rate and an user fee to be charged for using infrastructure services under the pure private provision of infrastructure.*

2.2. Pure public provision of infrastructure

In the pure public provision of infrastructure, the government solely renders the infrastructure services. Therefore, in this model, we have three agents namely; the representative household, a firm producing finished goods and the government. We assume that the government charges user fees to the firms for the usage of infrastructure not to maximize its profit but to meet the expenses of producing infrastructure. In addition to earning revenue from the user fees, the government charges capital income tax for financing the cost of infrastructure production.

2.2.1 The Government

The government is mainly engaged in 3 activities under the pure public provision of infrastructure: (1) it provides the infrastructure goods and charges user fees for the usage of infrastructure; (2) it imposes capital income tax in order to finance its cost; (3) it also issues government bonds. Therefore, interest on the bonds adds to the debt burden of the government while tax revenue reduces the government debt. The bond accumulation function is given by (25).

$$\dot{B} \;=\; (1-\tau)rB - (T - E) \tag{25}$$

T is the tax revenue at time t and E is the public expenditure at time t.

$$T = \; \tau\, r\, uK + \mu G \tag{26}$$

$$E = r\,(1-u)K \tag{27}$$

Substituting the value of T and E in equation (25), we obtain the bond accumulation function.

$$\dot{B} = (1-\tau)rB - [\tau ruK + \mu\delta(1-u)K - r(1-u)K] \tag{28}$$

The rate of growth of bond is obtained from equation (28).

$$\frac{\dot{B}}{B} = (1-\tau)r - \frac{K}{B}[\tau ru + \mu\delta(1-u) - r(1-u)] \tag{29}$$

2.2.2 The Household Sector

The utility function of the representative household is given in equation (1). However, contrary to the pure private provision of infrastructure, the wealth/asset of the household is now defined as the sum of bond holding (B) and capital holding (K) under the pure public provision of infrastructure.

$$W = B + K \tag{30}$$

Total disposable wealth of a household over consumption expenditure is accumulated as wealth. The rate of accumulation of wealth is given by equation (30).

$$\dot{W} = (1 - \tau)rW - C \tag{31}$$

Where τ is the tax on capital income, r is the interest rate, C is the consumption at time t. The representative household maximizes the current-value Hamiltonian, subject to the wealth accumulation function of equation (31).

$$H_c = \gamma \ \ln C + \eta[(1 - \tau)rW - C] \tag{32}$$

The control variable is C. The first – order maximization condition is given in equation (33).

$$\eta = \frac{\gamma}{C} \tag{33}$$

The time derivative of the co-state variable is given by equation (34).

$$\frac{\dot{\eta}}{\eta} = \rho - (1 - \tau)r \tag{34}$$

Taking the log and derivative of equation (33), we obtain the growth rate equation under the pure public provision.

$$-\frac{\dot{C}}{C} = \frac{\dot{\eta}}{\eta} = \rho - (1 - \tau)r \tag{35}$$

Resorting to equation (30), we have,

$$\frac{\dot{W}}{W} = \frac{\dot{B}/B + \dot{K}/K \cdot K/B}{1 + K/B}$$

Since $\frac{\dot{B}}{B} = \frac{\dot{K}}{K} = g$, now substituting these values in the above equation, we get,

$$\frac{\dot{W}}{W} = g \tag{36}$$

Using equations (35), (36) and (12), the growth rate equation under the pure public provision is g.

$$\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{\eta}}{\eta} = \frac{\dot{W}}{W} = (1-\tau)r - \rho = (1-\tau)A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \rho = g \tag{37}$$

Resorting to equation (31), we obtain the value of $\frac{C}{K}$.

$$\frac{C}{K} = \left(\frac{B}{K} + 1\right)[(1-\tau)r - g] \tag{38}$$

2.2.3 Steady State Balanced growth

The steady state balanced growth equilibrium is defined as a situation when consumption, private physical capital and infrastructure capital grow at the same strictly positive constant growth rate, i.e; $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = \frac{\dot{B}}{B} = \frac{\dot{Y}}{Y} = g$. If $\frac{\dot{B}}{B}$ is constant, then $\frac{B}{K}$ is also constant. We obtain the value of $\frac{B}{K}$ by setting $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$.

$$\frac{B}{K} = \frac{\tau r u + \mu\delta(1-u) - r(1-u)}{\rho} \tag{39}$$

2.2.4 Zero Debt regime

In the zero debt regime, we assume that the government doesn't have any debt such that the government's tax revenue is equal to the total expenditure of the government. In other words, the government observes the balanced budget. In the zero debt regime, we set $T = E$.

$$r[1 - u(1+\tau)] = \mu\delta(1-u) \tag{40}$$

Substituting the value of r and u obtained from the demand function of firm 1 in the budget constraint of the government, we obtain the user fees to be charged under the pure public provision in the zero-debt regime.

$$\mu = \left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{\alpha-1} \quad (41)$$

Equating equation (41) with equation (9), we find the equilibrium value of u, under the pure public provision in the balanced budget regime.

$$u = \frac{A\,(1-\alpha)}{\alpha + A(1-\alpha)(\tau+1)} \quad (42)$$

The growth rate under the pure public provision in the zero debt regime is obtained after substituting the values of rate of interest and the user fee in equation (37).

$$g = (1-\tau)A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{\alpha} - \rho \quad (43)$$

Differentiating the growth rate equation (43) with respect to τ, the first-order condition is,

$$A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{(\alpha-1)}\left(-A^{\frac{1}{\alpha-1}}\ \alpha^{\frac{\alpha}{\alpha-1}}\delta\left(\frac{\alpha}{1-\alpha}\right) - \right.$$

$$\left. A^{\frac{1}{\alpha-1}}\alpha^{\frac{1}{\alpha-1}}\ \delta\,\tau\,(\alpha+1)\right) < 0 \quad (44)$$

Equation (44) implies that the growth maximizing tax rate is zero. Therefore, the optimal growth rate under the pure public provision in the zero debt regime after substituting the value of growth maximizing tax rate is obtained in equation (45).

$$g_{Public}^{\dot{B}=0} = A(1-\alpha)^{1-\alpha}\alpha^{\alpha}\delta^{\alpha} - \rho \quad (45)$$

Proposition 2: *The growth maximizing tax rate under the pure public provision of infrastructure is zero. It is optimal for the government to charge user fees instead of imposing a tax for financing infrastructure. The optimal growth rate under the pure public provision in the balanced budget regime is equal to the growth rate of the pure private provision of infrastructure.*

2.2.5 Constant Debt regime:

We assume that at steady state, the government provides the infrastructure in the constant debt regime, such that $\dot{B} = 0$ and also $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = g$ is constant and positive. When $\dot{B}$ is equal to zero, it does not necessarily imply that public debt is zero. The debt to GDP ratio and the debt to capital ratio are both positive, because the level of initial debt is, positive. Both decline over time and converge to zero in the long run.

Therefore, setting $\dot{B} = 0$, we find that the user fees under the pure public provision of infrastructure in the constant debt regime and in the zero debt regime are same. The growth rate and growth maximizing tax rate are also found to be the same. Because of this result, the growth rate, and optimal growth rate are also found to be same. These findings are illustrated in Appendix A1. In the constant debt regime, the debt to capital ratio becomes zero in the long run, therefore the results are same as in the zero debt regime.

Proposition 3: *Under the pure public provision of infrastructure, the user fee charged for infrastructure services and growth rates are same in the constant debt regime and the balanced budget (zero debt) regime. Also, the optimal growth rates are same in both the regimes.*

Proof: The proof of proposition 3 is shown in Appendix A1.

2.2.6 Permanent deficit regime

We assume that at the steady state balanced growth rate, in the permanent deficit regime, the debt is accumulating. Steady state balanced growth rate g is positive and constant, therefore all the variables grow at the same strictly positive constant rate, such that $\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{W}}{W} = \frac{\dot{B}}{B} = g$. The permanent deficit case is characterized by the public deficit, where the government debt grows at the same rate as all other endogenous variables in the long run.

For the sustainability of public debt in our model, we apply the primary surplus rule following Griener (2008), which states that the primary surplus relative to GDP is a positive function of the debt to GDP ratio. According to Greiner (2013), 'the economic rationale behind the rule is to make the debt ratio a mean-reverting process when the reaction of the primary surplus is sufficiently large, preventing the debt to GDP ratio from exploding'. There is also empirical evidence revealing that the governments follow such a rule of the primary surplus. For example, Bohn (1998) and Greiner et al. (2007) using OLS estimations have shown that this rule holds for USA and selected European countries, respectively. Also, Fincke and Greiner (2012) find that the reaction coefficient determining the response of the primary surplus to public debt is not a constant but time-varying with the average of that coefficient being strictly positive for some Euro area countries.

Hence, following Greiner (2008) and Kamaiguchi and Tamai (2012), we assume that the ratio of the primary surplus to gross domestic income ratio is a positive linear function of the debt to gross domestic income ratio with an intercept. The primary surplus ratio is given in equation (46).,

$$\frac{T-E}{Y} = \xi + \beta \frac{B}{Y} \tag{46}$$

''Where ξ, β are the real numbers and are constant. β determines how strongly the primary surplus reacts to changes in public debt. ξ determines whether the level of the primary surplus rises or falls with an increase in gross domestic income. If ξ is less than zero implies that primary surplus declines as GDP rises and the government increases it's spending with higher GDP. In this case of negative ξ, β must be sufficiently large. If β is sufficiently low, then the government must be a creditor for the economy to achieve sustained growth. If ξ is greater than zero implies that primary surplus rises as GDP increases. In this case, β must not be too large. A high β implies that the government does not invest sufficiently and it must be a creditor in order to finance its investment, in order to achieve sustained growth.'' (Greiner, 2008)

From equation (46),

$$T - E = \xi\, Y + \beta\, B \tag{47}$$

Substituting the values of T, E and Y in the above equation, we get the value of $\frac{B}{K}$ in the permanent deficit regime.

$$\frac{B}{K} = \frac{\tau\, r\, u - \xi\, A\, u^{1-\alpha}\delta^{\alpha}(1-u)^{\alpha} - (1-u)(r-\mu\delta)}{\beta} \tag{48}$$

The bond accumulation function for the sustainability of public debt is given in equation (49).

$$\dot{B} = (1-\tau)rB - (\xi\, Y + \beta\, B) \tag{49}$$

The rate of growth of bond in the pure public provision in the accumulating debt regime is obtained after substituting the value of Y, $\frac{K}{B}$ and $\frac{G}{K}$ in the above equation,

$$\frac{\dot{B}}{B} = (1-\tau)r - \frac{\xi\, Au^{1-\alpha}[\delta(1-u)]^{\alpha}\beta}{\tau\, r\, u - \xi\, A\, u^{1-\alpha}\delta^{\alpha}(1-u)^{\alpha} - (1-u)(r-\mu\delta)} - \beta \tag{50}$$

Resorting to the growth rate equations (37) and (50), we set $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$ and hence obtain the user fees under the pure public provision in the accumulating debt regime.

$$\mu = \frac{\delta^{\alpha-1}\alpha}{A}\left[\tau - \left(\frac{(\rho-\beta)\alpha + \rho\xi}{(1-\alpha)(\rho-\beta)}\right)\right]^{\alpha-1} \tag{51}$$

Equating equation (51) with equation (9), we find the equilibrium value of u under the pure public provision in the accumulating debt regime.

$$u = \frac{(1-\alpha)(\rho-\beta)A^{\frac{2}{\alpha-1}}}{(\rho-\beta)\left[(1-\alpha)\left(\tau + A^{\frac{2}{\alpha-1}}\right) - \alpha\right] - \rho\xi} \tag{52}$$

The growth rate under the pure public provision in the permanent deficit regime is obtained after substituting the values of rate of interest and the user fees under the pure public provision in the permanent deficit regime.

$$g = A^{\frac{1+\alpha}{1-\alpha}}(1-\tau)(1-\alpha)\delta^{\alpha}\left[\tau - \left(\frac{(\rho-\beta)\alpha+\rho\xi}{(\rho-\beta)(1-\alpha)}\right)\right]^{\alpha} - \rho \tag{53}$$

We differentiate the growth rate equation (53) with respect to τ. The first order condition gives the optimal tax rate under the pure public provision in the accumulating debt regime.

$$\hat{\tau} = \frac{(\rho-\beta)\alpha+\rho\xi}{(1-\alpha)(\rho-\beta)(\alpha+1)} + \frac{\alpha}{(\alpha+1)} \tag{54}$$

The optimal tax rate must lie between zero and one. The conditions for optimal tax rate to be positive are given in equation (55) and (56). Equation (55) gives the condition for optimal tax rate to be greater than zero and equation (56) gives the condition for optimal tax rate to be less than one.

$$\alpha(2-\alpha) < \frac{\rho\xi}{(\rho-\beta)} \tag{55}$$

$$\frac{\rho\xi}{(\rho-\beta)} > \alpha \tag{56}$$

When the value of $\xi > 0$ such that the value of β is sufficiently small for the sustainability of public debt and $\alpha > 0.5$, the second order condition is satisfied.

$$A^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)\delta^{\alpha}\alpha\left[\frac{(\rho-\beta)\alpha(1-2\alpha)-\rho\xi\alpha}{(1-\alpha)(\rho-\beta)(\alpha+1)}\right]^{\alpha-1}\left[-2+\frac{(\alpha-1)}{\alpha}\right] < 0 \tag{57}$$

The optimal growth rate under the pure public provision in the accumulating debt regime is obtained by substituting the value of optimal tax rate in the growth rate equation (53).

$$g_{\hat{\tau}} = \left[\frac{(\rho-\beta)(1-2\alpha)-\rho\xi}{(\rho-\beta)(\alpha+1)}\right]^{1+\alpha} A^{\frac{1+\alpha}{1-\alpha}}\delta^{\alpha}\left(\frac{\alpha}{(1-\alpha)}\right)^{\alpha} - \rho \tag{58}$$

For the positive growth rate and existence of optimal tax rate $\alpha < \frac{1}{2} - \frac{\rho\xi}{2(\rho-\beta)}$ and the primary surplus is positive such that coefficient of public debt to GDP is sufficiently small.

Proposition 4: *For higher technology and output elasticity of infrastructure less than* $\alpha < \frac{1}{2} - \frac{\rho\xi}{2(\rho-\beta)}$ *then the optimal growth rate under the accumulating debt regime is greater than the optimal growth rate under the balanced budget regime of the pure public provision of infrastructure.*

Proof: Comparing the optimal growth rates of the pure public provision of infrastructure in the accumulating debt regime and the balanced budget regime. We find that,

$$g_{\hat{\tau}}^{\dot{B}>0} - g_{\hat{\tau}}^{\dot{B}=0} = \left[\frac{(\rho-\beta)(1-2\alpha)-\rho\xi}{(\rho-\beta)}\right]^{\alpha+1} > \frac{(1-\alpha)(\alpha+1)^{\alpha+1}}{A^{\frac{2\alpha}{1-\alpha}}} \tag{59}$$

Under the pure public provision for higher technology and $\alpha > \frac{1}{2} - \frac{\rho\xi}{2(\rho-\beta)}$ the optimal growth rate in the accumulating debt regime is greater than the optimal growth rate in the balanced budget regime. We find that, output elasticity of infrastructure capital is less than 0.5 and primary surplus is positive, such that coefficient of public debt is sufficiently small for the sustainability of public debt.

2.3. Public-Private Partnership provision of infrastructure

In the Public-Private Partnership provision of infrastructure the government provides the infrastructure services with the help of a private firm. However, in the partnership venture the ownership lies with the private firm and the government makes a small partial investment for manufacturing infrastructure. In real life, there are several PPP contracts such as Build-Operate-Transfer (BOT), Build-Own-Operate-Transfer

(BOOT), Design-Build-Finance-Operate (DBFO) and Design-Construct-Maintain-Finance (DCMF) etc. In countries, where PPP projects are implemented, there is collaboration between the government and the private firm for rendering infrastructure services. Through Viability Gap Funding (VGF), the government provides the capital grants to the infrastructure- manufacturing firm for the construction of infrastructure projects.

In this section, we construct a model where the government makes a partial investment in the private firm. Therefore, following the real-life examples of VGF in the infrastructure projects of the PPP mode, it is assumed that the government bears $(1-\phi)$ fraction of the total cost of manufacturing infrastructure, $(1-u)K$ fractions of the infrastructure. In this model, we have four agents namely; the representative household, two firms and the government. Now, first we look at the manufacturing sector, followed by the government sector and then the household sector.

Like the pure private provision case, here also we assume that firm 1 produces final goods for consumption and capital accumulation. Therefore, the production function of final goods is same as equation (2). The profit function of firm 1 is given in equation (5). In the PPP provision of infrastructure, firm 2 produces the infrastructure services, charges user fees for it and takes care of the partial cost of production of infrastructure. Infrastructure is used for final goods production and the welfare enhancement. The production function of infrastructure services is same as equation (3) and the profit function of firm 2 producing the flow of infrastructure service is given by (60).

$$\Pi^2 = \mu G - \phi\, r\, (1-u)K \tag{60}$$

In equation (60), μ is the user fee and ϕ is the share of cost borne by firm 2 for manufacturing infrastructure. Substituting the value of G in the above equation.

$\Pi^2 = \mu\delta(1-u)K - \phi\, r\,(1-u)K$

In the PPP provision of infrastructure as well, both firms take input prices as given and choose input quantities so as to maximize their profit. Differentiating the profit function of firm 1 with respect to uK and G, we obtain the first-order conditions same as equations (6) and (8). Differentiating the profit function of firm 2 with respect to $(1-u)K$, we obtain the rate of interest under the PPP provision.

$$r = \frac{\mu\delta}{\phi} \tag{61}$$

By no arbitrage condition, equating the rate of interest of firm 1 and firm 2, we obtain the user fee under the PPP provision.

$$\mu = \frac{\delta^{\alpha-1}\alpha^{\alpha}A}{\phi^{\alpha-1}(1-\alpha)^{\alpha-1}} \tag{62}$$

In equation (62), we obtain the user fee under the PPP provision of infrastructure.

2.3.1 The Government

The government is engaged in 3 activities under the PPP provision of infrastructure as well, like the pure public provision of infrastructure. However, unlike the pure public provision, it bears the partial cost of manufacturing infrastructure by investing in firm 2 and therefore, the total production cost of infrastructure do not fall on the government. Therefore, listing the 3 activities of the government under the PPP mode of provision, it includes: (1) bearing the partial cost of manufacturing infrastructure, (2) imposing capital income tax in order to finance its cost and (3) issuing government bonds.

Hence, the government in this economy receives fund by imposing capital income tax and by issuing government bonds. However, here in the PPP provision, the tax revenue and public expenditure equations are different from the pure public provision's tax revenue and public expenditure.

$$T = \tau\, r\, uK \tag{63}$$

$$E = (1-\phi)r\,(1-u)K \tag{64}$$

$(1-\phi)$ in the above equation is the share of cost of infrastructure manufacturing, borne by the government. Substituting the value of T and E in equation (25), the bond accumulation function of the government under the PPP provision is given in equation (65).

$$\dot{B} = (1-\tau)rB - \tau ruK + (1-\phi)r(1-u)K \tag{65}$$

From equation (65), we obtain the rate of growth of bond under the PPP provision of infrastructure.

$$\frac{\dot{B}}{B} = (1-\tau)r - \frac{K}{B}[\tau ruK - (1-\phi)r(1-u)K] \tag{66}$$

After looking at the manufacturing and the government sector, we now look at the household sector.

2.3.2 The Household sector

The utility function of the representative household is given in equation (1). The wealth/asset of the household and the rate of accumulation of wealth under the PPP provision remains same as the pure public provision of infrastructure, illustrated by equations (30) and (31). The representative household maximizes the current-value Hamiltonian, subject to the wealth accumulation function of equation (31). The first-order maximization condition gives same result as equation (33). Taking the time-derivative of the co-state variable and also taking the log and derivative of equation (33), we obtain the growth rate equation of the PPP provision of the infrastructure.

$$\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{\eta}}{\eta} = \frac{\dot{W}}{W} = (1-\tau)r - \rho = (1-\tau)\frac{\mu\delta}{\phi} - \rho = g \tag{67}$$

2.3.3 Steady State Balanced growth

The steady state balanced growth equilibrium is defined as a situation when consumption, private physical capital and infrastructure capital grow at the same strictly positive constant growth rate, i.e; $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = \frac{\dot{B}}{B} = \frac{\dot{Y}}{Y} = g$. Where g is positive and constant. If $\frac{\dot{B}}{B}$ is constant, then $\frac{B}{K}$ is also constant. We obtain the ratio of public debt to physical capital under the PPP provision by setting $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$.

$$\frac{B}{K} = \frac{\tau r u - (1-\phi) r (1-u)}{\rho} \tag{68}$$

Next, we evaluate the provision of infrastructure under the PPP in different budgetary regimes just like the provision of infrastructure under the pure public provision.

2.3.4 Zero Debt regime

In the zero debt regime for the PPP provision also we set tax revenue equation (63) equal to the public expenditure equation (64). We obtain the value of user fees under the PPP provision by setting $T = E$.

$$\mu = \frac{\tau^{\alpha-1} A\, \delta^{\alpha-1} \alpha}{(1-\phi)^{\alpha-1}} \tag{69}$$

Equating equation (69) with equation (62), we find the equilibrium tax rate under the PPP provision in the balanced budget regime.

$$\tau^* = \frac{\alpha(1-\phi)}{\phi(1-\alpha)} \tag{70}$$

We obtain the equilibrium growth rate under the PPP provision in the balanced budget regime by substituting the value of user fee and equilibrium tax rate.

$$g_{PPP}^{\dot{B}=0} = \frac{A \alpha^{\alpha} \delta^{\alpha} (\phi - \alpha)}{\phi^{\alpha+1} (1-\alpha)^{\alpha}} - \rho \tag{71}$$

2.3.5 Constant Debt regime

We also look at the PPP provision in the constant debt regime. By setting $\dot{B} = 0$, we find that the user fee, equilibrium tax rate and equilibrium growth rate obtained in the constant debt regime is exactly the same as the zero debt regime, just like the pure public provision case. These derivations are illustrated in Appendix A2.

Proposition 5: *Under the PPP provision in the balanced budget regime and the constant debt regime, a unique, steady state equilibrium growth rate, an equilibrium tax rate and user fee exists. Also, the equilibrium growth rate, tax rate and user fees are exactly same in both the constant debt and zero debt regimes.*

Proof: The proof for proposition 5 is shown in Appendix A2.

2.3.6 Permanent deficit regime

In this section, we look at the PPP provision of infrastructure in the permanent deficit regime. The steady-state balanced growth rate g is positive and constant in the permanent deficit regime. Therefore, all the variables grow at the same strictly positive constant rate, such that $\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{\dot{W}}{W} = \frac{\dot{B}}{B} = g$. Applying the primary surplus rule, we obtain the value of $\frac{B}{K}$ the permanent deficit regime.

$$\frac{B}{K} = \frac{\tau\, r\, u - \xi\, A\, u^{1-\alpha}\delta^{\alpha}(1-u)^{\alpha} - (1-\phi)\, r(1-u)}{\beta} \tag{72}$$

The rate of growth of bond under the PPP provision in the permanent deficit regime is obtained by substituting the values of Y, $\frac{K}{B}$ and $\frac{G}{K}$ in the bond accumulation function equation (49).

$$\frac{\dot{B}}{B} = (1-\tau)r - \frac{\xi\, A u^{1-\alpha}[\delta(1-u)]^{\alpha}\beta}{\tau\, r\, u - \xi\, A\, u^{1-\alpha}\delta^{\alpha}(1-u)^{\alpha} - (1-\phi)\, r(1-u)} - \beta \tag{73}$$

To obtain the value of user fee under the PPP provision in the permanent deficit regime, we set $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$.

$$\mu = \frac{[\tau\delta(\rho-\beta)]^{\alpha-1}A\alpha^{\alpha}}{[\xi\rho\phi+(1-\phi)(\rho-\beta)]^{\alpha-1}} \tag{74}$$

Equating equation (74) with equation (62), we find the equilibrium tax rate under the PPP provision in the permanent deficit regime.

$$\tau^{*} = \frac{\xi\rho\phi+(1-\phi)(\rho-\beta)}{\phi(1-\alpha)(\rho-\beta)} \tag{75}$$

We obtain the equilibrium growth rate under the PPP provision in the permanent deficit regime by substituting the value of user fee from eqution (74) and by substituting the value of equlibrium tax rate from equation (75).

$$g_{PPP}^{\dot{B}>0} = \frac{A\delta^{\alpha}\alpha^{\alpha}}{\phi^{\alpha}(1-\alpha)^{\alpha}}\left[\phi(2-\alpha) - 1 - \frac{\xi\rho\phi}{(\rho-\beta)}\right] - \rho \tag{76}$$

Proposition 7: *If* $\frac{\xi\rho}{(\rho-\beta)} > \frac{\alpha+\alpha^{2}(2-\alpha)}{\phi^{2}}$ *then the equilibrium growth rate in the accumulating debt regime is more than the equilibrium growth rates in the balanced budget regime under the PPP provision.*

Proof: Comparing the optimal growth rates in the accumulating debt regime with the constant debt regime or balanced budget regime, we find the following condition.

$$g_{PPP}^{\dot{B}>0} - g_{PPP}^{\dot{B}=0} = \frac{\xi\rho}{(\rho-\beta)} > \frac{\alpha+\alpha^{2}(2-\alpha)}{\phi^{2}} \tag{77}$$

For lower output elasticity of infrastructure capital and higher share of cost of borne by the infrastructure-manufacturing private firm, the condition given in equation (77) is easily satisfied. Therefore, debt financing or permanent deficit regime is better than the balanced budget regime, because the growth rate in the permanent deficit regime is greater than the growth rate in the balanced budget regime under the PPP provision.

3. An optimal mode of provision of infrastructure:

We may be able to find the optimal mode of provision of infrastructure by comparing the optimal growth rates and user fees for the different modes of provision of infrastructure respectively. First, we compare the optimal growth rate under the pure public/private provision with the equilibrium growth rate under the PPP provision in the balanced budget or constant debt regime.

$$g_{Public}^{\dot{B}=0} - g_{PPP}^{\dot{B}=0} = (1-\alpha)^{\frac{1}{\alpha}} > \frac{1}{\phi} \tag{78}$$

If output elasticity of infrastructure is high and share of cost borne by the infrastructure-manufacturing firm is low, then the PPP provision in the balanced budget regime is better than the pure public provision in the balanced budget regime or pure private provision of infrastructure. Secondly, we compare the optimal growth rates in the accumulating debt regimes for the PPP provision and the public provision.

$$g_{PPP}^{\dot{B}>0} - g_{Public}^{\dot{B}>0} = \frac{(\rho-\beta)^{\alpha}[(\rho-\beta)[\phi(2-\alpha)-1]-\xi\rho\phi]}{[(\rho-\beta)(1-2\alpha)-\rho\xi]^{\alpha+1}} > \frac{\phi^{\alpha}A^{\frac{\alpha}{1-\alpha}}}{(\alpha+1)^{1+\alpha}} \tag{79}$$

For output elasticity of infrastructure lower than 0.5 or $\alpha < \frac{1}{2} - \frac{\rho\xi}{2(\rho-\beta)}$ and primary surplus is negative, such that coefficient of debt to GDP ratio is sufficiently large, then the above condition is satisfied. For lower output elasticity of infrastructure, the PPP provision in the permanent deficit regime is better than the pure public provision in the permanent deficit regime.

Proposition 8: *In balanced budget regime, if the share of manufacturing cost of infrastructure borne by the government is high, then the PPP provision of infrastructure yield higher growth rate compared to pure public provision or pure private provision of infrastructure.*

Proof: From equation (78), the condition $(1-\alpha)^{\frac{1}{\alpha}} > \frac{1}{\phi}$ is sufficient to prove proposition 8.

Proposition 9: *In the accumulating debt regime, if the output elasticity of infrastructure is low, then the PPP provision yield higher growth rate compared to the pure public provision.*

Proof: The condition given in equation (79) is sufficient to prove proposition 9.

Resorting to equation (77) and proposition 7 for the choice of balanced budget regime or accumulating debt regime, the accumulating debt is preferred over the balanced budget regime for the PPP provision if there is lower output elasticity of infrastructure and higher share of cost borne by the infrastructure-manufacturing private firm. And, resorting to equation (59) and proposition 4, the technological parameter is high and the output elasticity of infrastructure is low for the public provision of infrastructure, the better is the permanent deficit regime as compared to the balanced budget regime.

Now, we make a comparison of the user fees under PPP provision and public provision of infrastructure in the permanent deficit regime. Since, equations (59), (77) and (79) suggests permanent deficit regime is better for both public provision and PPP provision, therefore comparing the user fees under the public provision and PPP provision in the permanent deficit regimes, we may be able to obtain the optimal mode of provision of infrastructure.

$$\mu_{Public}^{\dot{B}>0} - \mu_{PPP}^{\dot{B}>0} = \frac{(\rho-\beta)(1-2\alpha)-\rho\xi}{(\rho-\beta)(1+\alpha)} > \frac{1}{\phi} \tag{80}$$

From equation (80) it follows that, when output elasticity of infrastructure is high and share of cost borne by the infrastructure manufacturing private firm is low then the user fees charged under a pure public provision of infrastructure in the permanent deficit regime is greater than the PPP provision in the permanent deficit regime. Thus, if the share of manufacturing cost borne by the private firm is low, which means the share of manufacturing cost of infrastructure borne by the government is high in PPP, user-fee charged under purely public provision is higher than that of PPP provision of infrastructure under the accumulating debt regime, because VGF brings down the user fees charged under the PPP mode. This justifies PPP from the consumer's utility perspective as well. The output elasticity for the infrastructure capital increases due to VGF financing therefore, the government could opt for the provision of infrastructure by PPP mode.

Proposition 10: *If the share of cost borne by the government for infrastructure provision is high, then user fee charged under PPP would be lower, which may give another reason to justify PPP.*

4. Conclusion:

This paper explains, the conditions under which the PPP mode of infrastructure provision is desirable in a model with non-rival but excludable infrastructure in a closed economy model. The model shows that if the share of manufacturing cost borne by the government is low or in other words, the share of manufacturing cost borne by the private firm is high, then the PPP provision of infrastructure yields higher growth rate. The PPP provision in the accumulating debt regime may require charging lower user fees as compared to the pure public provision in the accumulating debt regime. In the model, the PPP provision may be justified on that ground too. The model also shows that for the technologically rich countries debt financing may be desirable under the pure public provision of infrastructure. Also, we find that for lower output elasticity of infrastructure, PPP provision of infrastructure in the accumulating debt regime is better as to PPP provision in the balanced budget and public provision in the accumulating debt regime.

There is a vast literature that deals with financing problem of public goods through pure public provision of infrastructure. But, there is not a single paper on endogenous growth theory, which deals with the financing problem of public goods through PPP. This paper tries to answer the two questions: (1) why should the government go for the PPP for the infrastructure provision? (2) How should the government finance the cost of infrastructure production – (a) through imposition of tax (b) through bond financing and (c) through charging user fees? We find the conditions for which the equilibrium growth rates under the PPP mode of provision is better than the optimal growth rates under the public provision. Also, the user-fee under the PPP mode is less as compared to the pure public user fees in the accumulating debt regime because the VGF investment in the infrastructure-manufacturing firm by the government reduces the user fees. Hence, we may conclude that, if the budget is balanced, pure public provision/ private provision of infrastructure is better than the PPP provision of infrastructure. In the permanent deficit regime, user fee under the PPP provision is lower than the user fees under the public provision. Moreover, the growth rate under the PPP provision in the permanent deficit regime is higher than pure public provision in the permanent deficit regime and also higher than the PPP provision in the balanced budget regime.

However, like any other theoretical model, this model also has few limitations. This model does not contain many aspects of the reality - like the imperfect competition in the production of infrastructure, infrastructure as a stock variable etc. In our future research we aspire to find optimal fiscal policy by taking these aspects into the consideration. However, this paper contributes to the literature being the first one to incorporate the public private partnership in infrastructure provision in endogenous growth model with constant, zero and accumulating government debt regimes.

APPENDIX

Appendix A1. Under the pure public provision in the constant debt, $\dot{B} = 0$. Therefore, from equation (25), we have, $T - E = (1-\tau)rB$.

Substituting the value of T and E from above equations (26) and (27) in the above equation, we obtain the user fees.

$r\ [\tau uK - (1-u)K - (1-\tau)\ B] = -\mu\ \delta(1-u)K$

Substituting the value of r and u obtained from the demand function of firm1 in the budget constraint of the government.

$$A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\left[\frac{\tau(A\alpha)^{\frac{1}{\alpha-1}}\delta}{\mu^{\frac{1}{\alpha-1}}+(A\alpha)^{\frac{1}{\alpha-1}}\delta} - \left(\frac{\mu^{\frac{1}{\alpha-1}}}{\mu^{\frac{1}{\alpha-1}}+(A\alpha)^{\frac{1}{\alpha-1}}\delta}\right) - (1-\tau)\frac{B}{K}\right] =$$

$$-\mu\delta\left(\frac{\mu^{\frac{1}{\alpha-1}}}{\mu^{\frac{1}{\alpha-1}}+(A\alpha)^{\frac{1}{\alpha-1}}\delta}\right)$$

Since $\frac{B}{K}$ converges to zero in the long run, therefore from the above equation we obtain the value of user fees for the pure public provision under the constant debt regime.

$$\mu = \left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{\alpha-1} \qquad \text{A1.1}$$

Now equating equation A1.1 with equation (9), we find the value of u.

$$u = \frac{A(1-\alpha)}{\alpha + A(1-\alpha)(\tau+1)} \qquad \text{A1.2}$$

Substituting the user fee in the growth rate equation (37), we obtain the growth rate equation under the pure public provision in the constant debt regime.

$$g = (1-\tau)A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{\alpha} - \rho \qquad \text{A1.3}$$

Differentiating growth rate equation (A1.3) with respect to τ, the first order condition is,

$$A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{(\alpha-1)}\left(-A^{\frac{1}{\alpha-1}}\ \alpha^{\frac{\alpha}{\alpha-1}}\frac{\delta\alpha}{(1-\alpha)} -\right.$$

$$\left. A^{\frac{1}{\alpha-1}}\alpha^{\frac{1}{\alpha-1}}\ \delta\ \tau\ (\alpha+1)\right) < 0 \qquad \text{A1.4}$$

Equation (A1.4) implies that the growth maximizing tax rate is zero. Therefore, the optimal growth rate under the pure public provision in the constant debt regime is given in equation (A1.5).

$$g = A(1-\alpha)^{1-\alpha}\alpha^{\alpha}\delta^{\alpha} - \rho \qquad \text{A1.5}$$

Appendix A2. Under the PPP provision in the constant debt regime, we set $\dot{B} = 0$. Therefore, from equation (65), we have, $\tau ruK - (1-\phi)r(1-u)K = (1-\tau)rB$.

$$\tau\, u - (1-\phi)(1-u) = (1-\tau)\frac{B}{K}$$

Since $\frac{B}{K}$ converges to zero in the long run, therefore from the above equation we obtain the value of user fee under the PPP provision of infrastructure services in the constant debt regime.

$$\mu = \frac{\tau^{\alpha-1}A\,\delta^{\alpha-1}\alpha}{(1-\phi)^{\alpha-1}} \qquad \text{A2.1}$$

Also, the equilibrium tax rate under the PPP provision in the constant-debt regime is same as balanced budget regime.

$$\tau^* = \frac{\alpha(1-\phi)}{\phi(1-\alpha)} \qquad \text{A2.2}$$

We obtain the equilibrium growth rate under the PPP provision in the constant debt regime by substituting the value of user fee and the equilibrium tax rate, which is same as the balanced budget regime.

$$g_{PPP}^{\dot{B}=0} = \frac{A(\phi-\alpha)\alpha^{\alpha}\delta^{\alpha}}{\phi^{\alpha+1}(1-\alpha)^{\alpha}} - \rho \qquad \text{A2.3}$$

Comparing equations (69), (70) and (71) with equations A2.1, A2.2 and A2.3 respectively we find that endogenous variables for both the constant debt regime and balanced budget regime are equal for the PPP provision of infrastructure.

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